Blocking an Argument for Emergent Chance

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Abstract

Several authors have argued that non-extreme probabilities used in special sciences such as chemistry and biology can be objective chances, even if the true microphysical description of the world is deterministic. This article examines an influential version of this argument and shows that it depends on a particular methodology for defining the relationship between coarse-grained and fine-grained events. An alternative methodology for coarse-graining is proposed. This alternative methodology blocks this argument for the existence of emergent chances, and makes better sense of two well-known subjects of philosophical discussion: the Miners Puzzle and Simpson's Paradox.

Keywords: chance; coarse-graining; probability

1 Introduction

Probabilistic explanations play an important role in many special sciences. Many authors in philosophy of science—including Loewer (2001), Cohen and Callender (2009), Hoefer (2007, 2019), Sober (2010), Glynn (2010), Frigg and Hoefer (2015), List and Pivato (2015), and Hemmo and Shenker (2019)—argue that the probabilities used in these explanations at least sometimes deserve the title of *objective chances*. As is standard, the term 'objective chance' refers here to a non-extreme probability that represents an observer-independent uncertainty about whether some event will

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occur. Objective chances stand in contrast to subjective or epistemic probabilities, which represent some agent or group of agents' uncertainty about whether a given event will occur. Crucially, and most controversially, the authors cited above hold that special science probabilities can be objective whether or not the true fundamental physical theory of the world is deterministic or indeterministic. That is, special science objective chances can *emerge* out of a deterministic physical theory. Let us call this position *the emergent chance thesis*.

In this paper, I focus my attention on List and Pivato's version of the emergent chance thesis, as it is the most formally precise version of the argument.[1] I show that their argument depends on a particular formal account of *coarsening*, where coarsening is the process through which more fine-grained descriptions of events are mapped to more coarse-grained descriptions of those same events. In light of this finding, I put forward an alternative account of coarsening that blocks List and Pivato's argument for the emergent chance thesis. I argue that my account has a key advantage over List and Pivato's: unlike their account, mine allows us to articulate crucial aspects of both Kolodny and MacFarlane's (2010) "Miners Puzzle" and the statistical phenomenon known as "Simpson's Paradox". In light of these arguments, I conclude that if the fundamental physical theory of the world is deterministic, then one must either regard all special science probabilities as purely epistemic (and therefore not objective chances), or furnish an argument for the emergent chance thesis that does not depend on List and Pivato's account of coarsening.

Here is the plan for the remainder of this paper. In Section 2, I give a perspicuous reconstruction of List and Pivato's formal definitions of events, probabilities, and coarsening. This provides the necessary background for Section 3, where I present their argument for the emergent chance thesis. In Section 4, I present my rival account of coarsening, show how it blocks List and Pivato's argument for the emergent chance thesis, and demonstrate how it is better equipped to aid our reasoning with respect to the Miners Puzzle and in cases of Simpson's Paradox. In Section 5, I consider and respond to counterarguments to my view. In Section 6, I offer concluding remarks.

[1] This choice of emphasis leaves to one side arguments due to Loewer (2001), Frigg and Hoefer (2015) and Hoefer (2019) which claim that the emergent chance thesis is entailed by a Humean best-systems appraoch to laws of nature.

2 Background on Probability and Coarsening

2.1 Events as Sets of Possible Worlds

Let us begin with List and Pivato's account of possible worlds. They define a *possible world* as a function $\omega(\cdot) : T \to S$, where T is a set of linearly-ordered times and S is a set of states of the world.[2] Thus, a possible world is a specification of what happens in a world at every point in time. The set of all possible worlds is denoted Ω, and the algebra $\mathcal{A}_\Omega$ is a set of subsets Ω that is closed under intersection, union, and complement. Importantly, List and Pivato take 'the set of all possible worlds' to mean the set of all *nomologically* possible worlds, or the set of worlds compatible with some set of laws. Where S is the set of all physically possible states of the world, the relevant laws are taken to be the laws of physics.[3]

List and Pivato define *events* as sets of possible worlds, such that each element of the algebra $\mathcal{A}_\Omega$ is an event. To illustrate, let E be the event that Kennedy is assassinated on November 22, 1963 at 12:30pm in Dallas. In List and Pivato's framework, this means that E is the set of all possible worlds in which the time t denoting November 22, 1963 12:30pm is mapped to a state s that is consistent with Kennedy being assassinated in Dallas. Importantly, although the set E includes the actual world, it also includes other counterfactual worlds, e.g. worlds where Kennedy was assassinated by someone other than Oswald, worlds where the assassination led to a nuclear war, and worlds where, just as Kennedy was assassinated, a star in a distant galaxy went supernova. The upshot here is that events can be used to pick out sets of actual and counterfactual possible worlds that share certain properties, even if those worlds are different in other respects.

2.2 Probabilities

A probability function $p_{\omega,t}(\cdot)$ is a mapping $\mathcal{A}_\Omega \to [0, 1]$. That is, $p_{\omega,t}(\cdot)$ takes as its input a set of possible worlds and returns a value in the unit interval. Since an event is defined as a set of possible worlds, the probability function effectively assigns any event a value in the unit interval.

[2] There are three tangential points to note here. First, List and Pivato use the term 'history' rather than 'possible world', but explicitly acknowledge that the two terms are interchangeable. Second, in a more complex version of their view, described in List and Pivato (2019), List and Pivato define possible worlds as mappings from a set of Cartesian products of spatial and temporal locations into the set of states. Finally, although every possible world $\omega(\cdot)$ is a function, in what follows I repress the function notation $(\cdot)$ and instead just refer to a possible world as ω.

[3] Although it is tangential to my argument, List and Pivato (2019) provide a rigorous methodology for determining the laws in a set of possible worlds, one that makes use of symmetry transformations on the set Ω.

The probability function is assumed to satisfy the Kolmogorov axioms; it assigns value 1 to Ω, assigns value 0 to the empty set, and assigns a probability to any countable union of disjoint events according to a principle of countable additivity.[4] As indicated by the notation, a probability function $p_{\omega,t}(\cdot)$ is indexed to a particular world and time. This is necessary because the probability assigned to the same event can change depending on the world and time at which the probability is assigned. For instance, in the actual world at times prior to his assassination, the probability that Kennedy would be assassinated in Dallas on November 22, 12:30pm was very low. However, in the actual world at times after his assassination, the probability of his being assassinated at that time and place is 1. By contrast, in worlds where he is not assassinated, the probability of Kennedy's assassination on November 22, 12:30pm is very low prior to that time, and 0 after that time.[5]

2.3 Coarsening

The crucial part of List and Pivato's formalism is their account of coarsening. They define a coarsening function $\sigma(\cdot)$, which is a mapping $S \to \mathbb{S}$, where S is a fine-grained set of states and $\mathbb{S}$ is a coarse-grained set of states. Importantly, the function need not be injective or surjective.[6] This means that while every element of the fine-grained set of states S is mapped to some unique element of the coarse-grained set of states $\mathbb{S}$, it may be that more than one element of S is mapped to a single element of $\mathbb{S}$. This is in keeping with the claim that coarse-grained states are multiply realized by fine-grained states, meaning that multiple, distinct fine-grained states are consistent with a single coarse-grained state. Since $\sigma(\cdot)$ is a function, it is also in keeping with the claim that coarse-grained states supervene on fine-grained states; there can be no change with respect to the coarse-grained state without a corresponding change with respect to the fine-grained state.

On List and Pivato's account, when we coarsen the set of states, this coarsening percolates

[4] Formally, if $\{E_1, E_2, ...E_n\}$ is a set of disjoint events, then $p(\bigcup_{i=1}^n E_i) = \sum_{i=1}^n p(E_i)$.

[5] In contexts where the state space S is represented as a continuum (e.g., if each possible state of a world corresponds to a real number), one's choice of probability measure over the algebra of possible worlds can have significant consequences for how that probability measure represents the dynamics of the system under study. Similarly, choosing a conditional expectation functional with which to define conditional probabilities is a non-trivial issue in the context of continuous possibility spaces (see, for example, Gyenis and Rédei 2017). In what follows, I confine myself to discrete possibility spaces in order to advance my argument against List and Pivato's approach to coarsening, and to articulate my own preferred approach. Ultimately, issues regarding one's choice of probability measure and conditional expectation functional are endemic to *all* attempts to use probability theory to represent the dynamics of continuous systems, including many physical systems.

[6] It will become clear in what follows why coarsening functions need not be injective. Coarsening functions are not surjective only in cases where there are elements of the set of higher-level states $\mathbb{S}$ that are not instantiated in any possible worlds. In practice, such states can be removed from $\mathbb{S}$ without consequence.

throughout the other features of their formal system, so that worlds, events and probabilities are also coarsened. Here is how this works. We begin by coarsening the states according to the function $\sigma(\cdot) : S \to \mathbb{S}$. Next, each coarse-grained world ω is defined as a mapping $T \to \mathbb{S}$. In other words, a coarse-grained world is a mapping from times to the coarse-grained set of possible states. This yields a set of all nomologically possible coarse-grained worlds $\mathbb{\Omega}$. We then define an algebra $\mathbb{A}_{\mathbb{\Omega}}$ over the set $\mathbb{\Omega}$. A coarse-grained event $\mathbb{E}$ is a set of coarse-grained worlds in $\mathbb{\Omega}$, such that each $\mathbb{E}$ is an element of the algebra $\mathbb{A}_{\mathbb{\Omega}}$. Finally, let $\mathbb{p}_{\omega,t}(\cdot)$ be a coarse-grained probability function mapping the coarse-grained algebra $\mathbb{A}_{\mathbb{\Omega}}$ into the unit interval.

To illustrate this account of coarsening, consider a toy example. Suppose that in every nomologically possible world, all that exists is a six-sided die. Once each minute, the die rolls itself, such that it can land showing any of its six sides, including the side that was showing at the previous minute. Recall that each possible world ω is a mapping $T \to S$, where T is the set of times and S is a set of states. Let each state $s_i \in S$ be a number in the set $S = \{1, 2, 3, 4, 5, 6\}$, where the number corresponds to the side of the die that is facing up. The set of possible mappings of times into this set of states is the set of possible worlds Ω, on which we can define an algebra $\mathcal{A}_\Omega$. Let t_1 and t_2 be times in T such that t_1 and t_2 are one minute apart, and let E_{t_2} be the set of possible worlds in which the die is showing six at t_2. If we suppose that, at any given time t_1, the probability that the die will show six at t_2 is 1/6, then this can be coherently written in List and Pivato's formalism as $p_{\omega,t_1}(E_{t_2}) = 1/6$.

Now suppose that we want to coarsen the set of states, and so we map the fine-grained set of states $S = \{1, 2, 3, 4, 5, 6\}$ into the coarse-grained set of states $\mathbb{S}$={odd, even}. Let us define a coarsening function $S \to \mathbb{S}$ such that $\sigma(1)$=odd, $\sigma(2)$=even, $\sigma(3)$=odd, $\sigma(4)$=even, $\sigma(5)$=odd, and $\sigma(6)$=even. This yields a coarse-grained set of worlds $\mathbb{\Omega}$ and a coarse-grained event algebra $\mathbb{A}_{\mathbb{\Omega}}$ in which each possible world specifies, for each time, whether the number facing up on the die is odd or even. Let $\mathbb{E}_{t_2}$ be the set of possible worlds in which the die is showing an even number at t_2. If we suppose that, at any given time t_1, the probability that the die will show an even number at t_2 is 1/2, then this can be written in List and Pivato's formalism as $\mathbb{p}_{\omega,t_1}(\mathbb{E}_{t_2}) = 1/2$.

This completes the exposition of List and Pivato's formal framework. The important aspect of the framework to flag here is the way that coarsening is represented. Specifically, it is important to note that List and Pivato coarsen the set of states that the world can be in, and then it is this initial coarsening that implies a coarse-grained set of possible worlds, a coarse-grained algebra of events, and a coarse-grained probability function. In the next section, I will show how this strategy for representing coarsenings plays a key role in List and Pivato's argument for the emergent chance thesis. In subsequent sections, I will advance an alternative formal account of coarsening that has advantages over List and Pivato's account, and blocks their argument for the emergent chance thesis.

3 The Emergent Chance Thesis

Recall that the emergent chance thesis says that it is possible for special science probabilities to be objective chances even if a more fine-grained algebra of events presents an entirely deterministic picture of the dynamics of the natural world. Crucially, this is a claim about what is possibly the case, rather than what is actually the case. As such, proponents of the emergent chance thesis must only establish the possible consistency of coarse-grained objective chances and fine-grained determinism.

The first step in explicating the argument for the emergent chance thesis is to make it more precise. Although List and Pivato follow Schaffer (2007) in offering a wide-ranging discussion of the nature of objective chance, and explicating six desiderata for a concept of objective chance, their core argument for the emergent chance thesis begins with their statement of a sufficient condition for a probability to be *purely epistemic*. This condition is stated as follows.

> **Test for Purely Epistemic Probability**: For a set of possible worlds Ω, let $I_{\omega,t}$ be a set of possible worlds such that every $\omega' \in I_{\omega,t}$ is in the same state as ω up until time t. In world ω and time t, the probability $p_{\omega,t}(E)$ is purely epistemic if $p_{\omega,t}(E|I_{\omega,t}) = 1$ or $p_{\omega,t}(E|I_{\omega,t}) = 0$.[7] (2015, p. 131).

In other words, a non-extreme probability of some event is purely epistemic at a world ω and time t if conditionalizing on the entire history of ω until t renders the probability extreme.

A natural response to this test is to ask why an extreme value of the conditional probability $p_{\omega,t}(E|I_{\omega,t})$ is not *necessary* and sufficient for any non-extreme probability $p_{\omega,t}(E)$ to be purely

[7] The formalism here has been changed slightly, though not consequentially, from the original. This is for the sake of coherence with my exegesis above.

epistemic. I take it that the answer is as follows. Suppose that we assign unconditional probability .5 to a coin coming up heads. Suppose further that if we conditionalize on the full history of the world up until we flip the coin, the conditional probability that the coin comes up heads is .999. Under these conditions, there is still some indeterminism to the coin toss, but not much. List and Pivato's sufficient condition for a purely epistemic probability is not satisfied, but one might still want to claim that the initial assignment of .5 to the coin coming up heads is purely epistemic. Even though there is some indeterminism with respect to the outcome of the coin toss, the initial probability assignment of .5 to the coin coming up heads is solely a representation of ignorance about the coin coming up heads. Thus, a probability $p_{\omega,t}(E)$ can be purely epistemic even when $p_{\omega,t}(E|I_{\omega,t})$ is not extreme. On the other hand, if it is established that $p_{\omega,t}(E)$ is purely epistemic, then we know that it is at least possible that $p_{\omega,t}(E|I_{\omega,t}) = 1$.

Equipped with this sufficient condition for a purely epistemic probability, we can construct an argument for the emergent chance thesis from a formal possibility result:

> **Emergence of Non-Epistemic Probabilities**: It is possible that all non-extreme probabilities in a distribution defined over a fine-grained algebra $\mathcal{A}_\Omega$ are purely epistemic and that some non-extreme probabilities in a distribution defined over the coarse-grained algebra $\mathbb{A}_{\mathbb{\Omega}}$ are not purely epistemic.

This claim can be restated as follows. Let Ω be a set of possible worlds such that each world ω is a mapping $T \to S$. $\mathcal{A}_\Omega$ is the event algebra on Ω. Let it be the case that, for any event $E \in \mathcal{A}_\Omega$, any world ω, and any time t, $p_{\omega,t}(E|I_{\omega,t}) = 1$ or $p_{\omega,t}(E|I_{\omega,t}) = 0$, where the event $I_{\omega,t}$ retains its prior interpretation. Thus, any output of $p_{\omega,t}(\cdot)$ is purely epistemic. All of these assumptions are consistent with a coarsening function $\sigma(\cdot) : S \to \mathbb{S}$ that yields the coarse-grained set of possible worlds $\mathbb{\Omega}$, the event algebra $\mathbb{A}_{\mathbb{\Omega}}$, and the probability function $\mathbb{p}_{\omega,t}$ such that for some coarse-grained events $\mathbb{E}$ and $\mathbb{I}_{\omega,t}$, $\mathbb{p}_{\omega,t}(\mathbb{E}|\mathbb{I}_{\omega,t}) \in (0,1)$. This establishes the emergence of non-epistemic probabilities.

To show that this possibility result holds, consider the following example (List and Pivato 2015, p. 136). A set of possible worlds Ω is defined so that each world ω is a function from times to states denoted by real numbers in the interval $[0,1]$. The history of each world evolves according to a transition rule such that if ω is in state s at t, then ω is in state $f(s)$ at $t+1$, where $f(s)$ is defined as follows.

$$f(s) = \begin{cases} 2s & \text{if } 0 \le s \le 1/2 \\ 2-2s & \text{if } 1/2 < s \le 1 \end{cases} \tag{1}$$

In this system, any probabilities defined over the algebra $\mathcal{A}_\Omega$ are purely epistemic. Given some specification of the prior states of any world at any time t, the subsequent evolution of that world is a matter of certainty, such that for any ω, t, and E, $p_{\omega,t}(E|I_{\omega,t}) = 1$ or $p_{\omega,t}(E|I_{\omega,t}) = 0$. For instance, if I_{ω,t_1} is the set of worlds such that $\omega(t_1) = 1/7$, and E is the set of worlds such that $\omega(t_2) = 2/7$, then $p_{\omega,t_1}(E|I_{\omega,t_1}) = 1$.

However, suppose that we map the fine-grained set of states denoted by real numbers in $S = [0, 1]$ into a coarse-grained set of states $\mathbb{S} = \{A, B\}$, via the following function:

$$\sigma(s) = \begin{cases} A & \text{if } 0 \le s \le 1/2 \\ B & \text{if } 1/2 < s \le 1 \end{cases} \tag{2}$$

Applying this coarsening function means that if any world is in the fine-grained set of worlds I_{ω,t_1}, i.e. the set of worlds such that $\omega(t_1) = 1/7$, then it is mapped into the coarse-grained set of worlds $\mathbb{I}_{\omega,t_1}$, which is the set of worlds such that $\omega(t_1) = A$. Similarly, if any world is in the fine-grained set of worlds E, i.e. the set of worlds such that $\omega(t_2) = 2/7$, then it is mapped into the coarse-grained set of worlds $\mathbb{E}$, which is the set of worlds such that $\omega(t_2) = A$. However, there is no inconsistency in it being the case that $\mathbb{p}_{\omega,t_1}(\mathbb{E}|\mathbb{I}_{\omega,t_1}) \in (0, 1)$. To see why, observe that there are many coarse-grained worlds ω such that $\omega(t_1) = A$ and $\omega(t_2) = B$. Namely, they are just those coarse-grained worlds $\omega \in \mathbb{I}_{\omega t_1}$ that are coarsenings of fine-grained worlds ω in which $\omega(t_1) \in (1/4, 1/2]$. Thus, even though the fine-grained probability distribution $p_{\omega,t}(\cdot)$ is purely epistemic for any ω, t, and E, the coarse-grained probability distribution $\mathbb{p}_{\omega,t_1}$ that emerges as the result of applying the coarsening function $\sigma(\cdot)$ to each world in Ω may not be purely epistemic for all ω, t, and $\mathbb{E}$.

If we accept the minimal assumption that only a non-extreme, not-purely-epistemic probability can represent an objective chance, and that some non-extreme, not-purely-epistemic probabilities do represent objective chances, then the emergence of non-epistemic probabilities directly entails the following:

Emergent Chance Thesis: It is possible that all probabilities in a distribution defined over a fine-grained algebra $\mathcal{A}_\Omega$ do not represent objective chances, and that some probabilities in a distribution defined over the coarse-grained algebra $\mathbb{A}_{\mathbb{\Omega}}$ do represent objective chances.

If we let Ω be the set of possible worlds described by some true microphysical theory of the world (i.e. the set of mappings from times to states that specify all physical properties of a world at a given time), and let $\mathbb{\Omega}$ be the set of possible worlds described by some special science (e.g. the set of mappings from times to states that specify all biological properties of a world at a given time), then it is clear how this formal statement of the emergent chance thesis establishes the informal statement of the thesis given in the introduction.

List and Pivato's possibility result depends crucially on their formal way of representing coarse-graining. To see this, consider the following potential response to the case given above. As before, let $\mathbb{E}$ be the set of coarse-grained worlds such that $\omega(t_2) = A$, and let I_{ω,t_1} be the fine-grained set of worlds such that $\omega(t_1) = 1/7$. It would seem that $\mathbb{p}_{\omega,t_1}(\mathbb{E}|I_{\omega,t_1}) = 1$, i.e. that the probability that a world is such that $\omega(t_2) = A$, given that it is in the fine-grained state $\omega(t_1) = 1/7$, is 1. If this were true, then non-extreme probabilities at both the coarser and finer levels of granularity would both be purely epistemic, and the example would not serve to establish the emergence of non-epistemic probabilities. This counterargument is a formalization of a broader line of response that proceeds as follows. If, according to a fine-grained description of the world, all non-extreme probabilities are purely epistemic, then all non-extreme probabilities assigned to events described at a more coarse-grained level must be purely epistemic. This is because non-extreme probabilities assigned to coarse-grained events reflect ignorance about the fine-grained details of the world.

List and Pivato's response to this line of counterargument is to point out that the probability $\mathbb{p}_{\omega,t_1}(\mathbb{E}|I_{\omega,t_1})$ is not mathematically well-defined. In any conditional probability, the conditioning event and the event being assigned a probability must be in the same algebra of events, which $\mathbb{E}$ and $I_{\omega,t}$ clearly are not (2015, p. 135). Thus, List and Pivato use their formal account of coarsening to derive what Hemmo and Shenker refer to as an *inaccessibility* relation between macro-level and micro-level descriptions of a possible world (2019, p. 469-470). This argumentative move on List and Pivato's part poses a challenge for anyone who aims to counter the emergent chance thesis. Namely, in order to block the emergent chance thesis, we need a formal account of coarsening that satisfies the following three argumentative desiderata: 1) the account must allow for the supervenience and multiple realization relations to hold between events described at finer and coarser levels of granularity, 2) the account must represent fine-grained and coarse-grained events as elements in the same algebra, and 3) the account must have further advantages beyond simply blocking the emergent chance thesis. I clarify that these three conditions are desiderata for an *argument* that blocks the emergent chance thesis, rather than desiderata for any account of higher-level probability. If these conditions were desiderata in the latter sense, then I would be begging the question with respect to List and Pivato's argument, since they reject the second desideratum. In the next section, I present an account of coarsening that satisfies all three of these desiderata.

4 An Alternative Account of Coarsening

4.1 The Account

The basic formal machinery of my account is unchanged from List and Pivato's. Like them, I define possible worlds as mappings from times to states, and define probability distributions as mappings from the algebra of possible worlds to the unit interval, with each distribution indexed to a particular time and world. Where my account differs from List and Pivato's is in the definition of the coarsening function. Whereas they define their coarsening function $\sigma(\cdot)$ as a mapping from a fine-grained set of states S to a coarse-grained set of states $\mathbb{S}$, I define a coarsening function $\phi(\cdot) : \mathcal{A}_\Omega \to \mathcal{A}_\Omega$, i.e. a mapping from the event algebra into itself. Importantly, $\phi(\cdot)$ has to satisfy the constraint that, for any event $E \in \mathcal{A}_\Omega$, it is the case that $E \subseteq \phi(E)$. That is, any event in the algebra must contain only worlds that are also in its coarsening. This constraint is important since it ensures that any fine-grained state is logically consistent with, though not necessarily equivalent to, its own coarsening. Since $\phi(\cdot)$ is a function, supervenience holds; there can be no change at the coarse-grained level without a change at the fine-grained level. Since $\phi(\cdot)$ is possibly not injective, coarse-grained events can be multiply realized by fine-grained events. Thus, my account satisfies the first of the desiderata listed above.

To illustrate how this approach to coarsening works, consider again the simple example of a set of worlds that each contain a single die that repeatedly rolls itself. Each world ω is a mapping from times to a set of states $S = \{1, 2, 3, 4, 5, 6\}$, where the number designating the state corresponds to the side of the die that is facing up. Let E_{ω,t_2} be the event that the die shows a six at t_2, so that $p_{\omega,t_1}(E_{\omega,t_2}) = 1/6$. Now suppose that we want to represent the event that the die shows an even number at t_2. In List and Pivato's framework, this requires us to map each element of the fine-grained set of states $S = \{1, 2, 3, 4, 5, 6\}$ into the coarse-grained set of states $\mathbb{S} = \{\text{odd}, \text{even}\}$. In my framework, we map E_{ω,t_2} into its coarsening $\phi(E_{\omega,t_2})$, where $\phi(E_{\omega,t_2})$ denotes the set of possible worlds in which $s = 2$, $s = 4$, or $s = 6$ at t_2. On this account of coarsening, as in List and Pivato's, we can coherently claim that the probability that the die will show an even number at t_2 is 1/2, i.e. that $p_{\omega,t_1}(\phi(E_{\omega,t_2})) = 1/2$.

Importantly, my account of coarsening has the same representational power as List and Pivato's. That is, any functional relationship between coarse-grained and fine-grained events that can be represented using their coarsening function $\sigma(\cdot)$ can also be represented by my coarsening function $\phi(\cdot)$. To make this claim more precise, let $\mathcal{A}_\Omega$ be an algebra on the set of possible worlds. Let $\sigma(\cdot)$ be a coarsening function mapping the fine-grained set of states S into the coarse-grained set of states $\mathbb{S}$. If $\mathbb{A}_\mathbb{\Omega}$ is the algebra of coarse-grained events that results from applying the coarsening $\sigma(\cdot)$ to the set of states S, let us say that $\mathbb{A}_\mathbb{\Omega}$ is implied by $\sigma(\cdot)$. Finally, let $\Gamma_{\phi(\cdot)}$ be the range of some coarsening function $\phi(\cdot)$, where $\phi(\cdot)$ is a coarsening function mapping $\mathcal{A}_\Omega$ into itself.[8] The following proposition is true:

Proposition 1. *For any Ω and any $\mathbb{A}_\mathbb{\Omega}$ implied by some coarsening function $\sigma(\cdot)$, there is a coarsening function $\phi(\cdot)$ such that there is a bijection between $\Gamma_{\phi(\cdot)}$ and $\mathbb{A}_\mathbb{\Omega}$.*

In other words, for any coarsening produced by List and Pivato's method, we can produce a coarsening via my method that is capable of representing the same coarse-grained events. Thus, any relations of supervenience and multiple realization that are represented by List and Pivato's coarsening function can also be represented via my coarsening function.

4.2 Blocking the Emergent Chance Thesis

It should be clear that this account of coarsening also satisfies the second desideratum for a response to List and Pivato, namely that coarse-grained and fine-grained events are represented as elements

[8] That is, $\mathcal{A}_\Omega$ is the codomain of the coarsening function $\phi(\cdot)$, and $\Gamma_{\phi(\cdot)}$ is the range of the function $\phi(\cdot)$.

of the same algebra. Given the extent to which the emergent chance thesis depends on representing coarse-grained and fine-grained events as elements of a different algebra, this formal result suffices to block the argument for the emergent chance thesis. To illustrate, consider the example of emergent chances given above, in which fine-grained states are represented by real numbers in the unit interval, worlds evolve temporally according to the function described by equation (1), and the set of states is coarsened according to the function described by equation (2). On List and Pivato's framework, the probability that the world is in coarse-grained state A at t_2, given that it is in coarse-grained state A at t_1, is not extreme, and therefore the probability that the world is in coarse-grained state A at t_2 is not purely epistemic. This is meant to hold even though the world's being in coarse-grained state A at t_2 is a logical consequence of the world's being in fine-grained state 2/7 at t_2, and even though any non-extreme probability assigned to the world's being in fine-grained state 2/7 at t_2 *is* purely epistemic. Such a result can only hold if we take the fine-grained and coarse-grained events to be elements of a different algebra. As my account represents coarse-grained and fine-grained events as elements of the same algebra, we get the result that if any non-extreme probability assigned to the world's being in fine-grained state 2/7 at t_2 is purely epistemic, then any probability assigned to the world's being in coarse-grained state A at t_2 is also purely epistemic, a result at odds with the emergence of objective chances in this case.

4.3 Further Advantages: The Miners Puzzle

So far, we have two different strategies for representing events at different levels of granularity: List and Pivato's, which entails the emergent chance thesis, and mine, which blocks their argument for the emergent chance thesis. In order to motivate my account, I need to provide some evidence that it has advantages over List and Pivato's. In what follows, I provide two cases that I believe constitute such evidence. Specifically, I show that my account of coarsening allows us to make better sense of two cases that are well known in the philosophy literature: the Miners Puzzle and Simpson's Paradox.

I begin with the Miners Puzzle, which is introduced by Kolodny and MacFarlane (2010). The puzzle imagines a scenario in which ten miners are trapped down one of two mine shafts: shaft A or shaft B. All of the miners are in the same shaft, whether that is shaft A or shaft B. A rainstorm is coming, and rescuers outside the mine have three options. They can block shaft A, block shaft

	Miners in Shaft A	Miners in Shaft B
Block A	10 Saved	0 Saved
Block B	0 Saved	10 Saved
Block Neither	9 Saved	9 Saved

Table 1: Decision Problem in Miner's Puzzle

B, or block neither shaft. Blocking one shaft results in the other shaft being completely full of water, while blocking neither shaft results in both shafts being only partially full of water. So, if the miners are in shaft A, then blocking shaft A will save them all, but if they are in shaft B, then blocking shaft A will drown them all. Blocking neither shaft will result in the drowning of one miner, whether the miners are in shaft A or B.

The decision problem facing rescuers can be represented via Table 1. Given the structure of the problem, the number of expected lives saved by each action can be calculated by taking the probability that a given action saves any lives, and multiplying it by the number of lives saved. So if the miners are equally likely to be in either shaft, then the expected lives saved by blocking shaft A is $.5(10) = 5$, and the same is true for blocking shaft B. By contrast, the expected lives saved from blocking neither shaft is $1(9) = 9$. So, by consequentialist lights, rescuers should not block either shaft, and thereby save nine lives. Kolodny and MacFarlane go on to argue that on *any* ethical framework, rescuers should block neither shaft.

The Miners Puzzle poses a challenge for accounts of normative decision. It is true that if the miners are in shaft A, then rescuers ought to block A, and the same is true of shaft B. Further, it is stipulated that the miners are either in shaft A or B, and thus it follows that rescuers should either block shaft A or block shaft B. However, it is clear that what they really ought to do is block neither shaft. My aim here is not to address this puzzle directly, but rather to show how List and Pivato's approach to coarsening hamstrings our ability to articulate the subtleties of the puzzle. A crucial fact about the puzzle is as follows. When the rescuers assign probability .5 to the event that blocking either shaft will save lives, this probability is purely epistemic. If rescuers learn that the miners are in shaft A, for instance, then the probability that blocking shaft A saves ten lives goes to 1, and so blocking this shaft becomes more prudent than blocking neither shaft. The same goes for blocking shaft B if the rescuers learn that the miners are in shaft B.

This can be cashed out using List and Pivato's formalism. Let S be the set containing all and only the following states:

- $s_1 =$ 10 miners are alive in shaft A and neither shaft is blocked.
- $s_2 =$ 10 miners are alive in shaft B and neither shaft is blocked.
- $s_3 =$ The rescuers block shaft A and 10 miners are alive in shaft A.
- $s_4 =$ The rescuers block shaft A and 10 miners are drowned in shaft B.
- $s_5 =$ The rescuers block shaft B and 10 miners are alive in shaft B.
- $s_6 =$ The rescuers block shaft B and 10 miners are drowned in shaft A.
- $s_7 =$ The rescuers block neither shaft and nine miners are alive in either shaft.

At t_1, the miners are either in shaft A or B, and the decision about whether to block either shaft is made at t_2. That is, at t_1, any world ω is in state s_1 or s_2, while at t_2, ω is in either s_3, s_4, s_5, s_6 or s_7. Let BA be the set of worlds such that rescuers block shaft A, i.e. worlds such that $\omega(t_2) \in \{s_3, s_4\}$. Let BB be the set of worlds such that rescuers block shaft B, i.e. worlds such that $\omega(t_2) \in \{s_5, s_6\}$, and let BN be the set of worlds such that the rescuers block neither shaft, i.e. worlds such that $\omega(t_2) \in \{s_7\}$. Finally, let X be the set of worlds in which any miners are saved, i.e. worlds such that $\omega(t_2) \in \{s_3, s_5, s_7\}$. For any world ω, $p_{\omega,t_1}(X|BA) = .5$, $p_{\omega,t_1}(X|BB) = .5$, and $p_{\omega,t_1}(X|BN) = 1$. However, to show that $p_{\omega,t_1}(X|BA) = .5$ and $p_{\omega,t_1}(X|BB) = .5$ are purely epistemic, let I_{ω,t_1} be the set of possible worlds that match ω up until t_1. Conditionalizing on this set specifies the state of the world at t_1; that is, it tells rescuers which shaft the miners are in. If the miners are in shaft A, then $p_{\omega,t_1}(X|BA, I_{\omega,t_1}) = 1$ and $p_{\omega,t_1}(X|BB, I_{\omega,t_1}) = 0$; the opposite holds if the miners are in shaft B. Thus, the non-extreme probabilities assigned to the event X, given either BA or BB, are purely epistemic.

This analysis allows us to calculate a response to the following important question: given that rescuers do not know how many miners are in either shaft, how much should they pay to learn this information? The answer is straightforward. In the absence of full information, they will block neither shaft and save nine lives. If they have full information, then they will block either shaft A or shaft B, and save ten lives. So they should incur costs up to the value of one miner's life in order to learn which shaft the miners are in. Using the formalism developed above, this result can be calculated as follows.

$$10 \cdot p_{\omega,t_1}(X|BA, I_{\omega,t_1}) + 10 \cdot p_{\omega,t_1}(X|BB, I_{\omega,t_1}) - 9 = 1 \tag{3}$$

To see the reasoning behind this calculation, note that whatever information is contained in I_{ω,t_1}, rescuers at t_2 will either block shaft A or block shaft B and thereby save ten lives, as compared to the nine lives they would have saved had they blocked neither shaft.

So far, so good for List and Pivato's analysis of the scenario. However, problems arise when we try to adapt their methodology for coarsening to this case. Let $\sigma(\cdot)$ be a coarsening function of the set of states S that is defined as follows.

$$\sigma(s_i) = \begin{cases} \mathbb{s}_{\mathbb{i}} & \text{if } i = 1 \\ \mathbb{s}_{\mathbb{i}-\mathbb{1}} & \text{if } i \neq 1 \end{cases} \tag{4}$$

In other words, any fine-grained state s_i is mapped to a coarse-grained state $\mathbb{s}_{\mathbb{i}-\mathbb{1}}$, except for s_1, which is mapped to $\mathbb{s}_{\mathbb{1}}$. Next, we give the following interpretation to all coarse-grained states:

- $\mathbb{s}_{\mathbb{1}}$ = 10 miners are alive in shaft A or shaft B, and neither shaft is blocked.
- $\mathbb{s}_{\mathbb{2}}$ = The rescuers block shaft A and 10 miners are alive in shaft A.
- $\mathbb{s}_{\mathbb{3}}$ = The rescuers block shaft A and 10 miners are drowned in shaft B.
- $\mathbb{s}_{\mathbb{4}}$ = The rescuers block shaft B and 10 miners are alive in shaft B.
- $\mathbb{s}_{\mathbb{5}}$ = The rescuers block shaft B and 10 miners are drowned in shaft A.
- $\mathbb{s}_{\mathbb{6}}$ = Rescuers block neither shaft and 9 miners are alive in either shaft.

Thus, the coarsening function $\sigma(\cdot)$ elides information about which shaft the miners are in at t_1. Let $\mathbb{BA}$ be the event in the resulting coarse-grained algebra such that rescuers block shaft A, i.e. the set of worlds such that $\omega(t_2) \in \{\mathbb{s}_{\mathbb{2}}, \mathbb{s}_{\mathbb{3}}\}$. Let $\mathbb{BB}$ be the event in the resulting coarse-grained algebra such that rescuers block shaft B, i.e. the set of worlds such that $\omega(t_2) \in \{\mathbb{s}_{\mathbb{4}}, \mathbb{s}_{\mathbb{5}}\}$. Let $\mathbb{X}$ be the event in the resulting coarse-grained algebra such that any miners are saved i.e. the set of worlds such that $\omega(t_2) \in \{\mathbb{s}_2, \mathbb{s}_4, \mathbb{s}_6\}$. The sufficient condition for a probability to be purely epistemic is not satisfied for $\mathbb{p}_{\omega,t_2}(\mathbb{X}|\mathbb{BA}) = .5$ and $\mathbb{p}_{\omega,t_2}(\mathbb{X}|\mathbb{BB}) = .5$ within this algebra. Conditionalizing on the history of the world up to t_2 will not tell rescuers which shaft the miners are in, and therefore will not assign an extreme probability to the event that blocking any shaft saves lives. Indeed, this coarsening function partitions the possible relevant states of the world into all and only those states that are currently epistemically accessible to the rescuers, thereby representing their limited knowledge of the location of the miners.

One upshot of this analysis is that the question 'what costs should the rescuers be willing to incur to learn what shaft the miners are in?' cannot be well-posed in this coarse-grained algebra. This is because no element of the coarse-grained algebra can specify which shaft the miners are in. Any attempt to do so would require conditionalizing on an element of a more fine-grained algebra to assign an event in a more coarse-grained algebra a conditional probability. In other words, we cannot answer this question because the following equation contains terms with no coherent mathematical definition:

$$10 \cdot \mathbb{P}_{\omega,t_2}(\mathbb{X}|\mathbb{BA}, I_{\omega,t_1}) + 10 \cdot \mathbb{P}_{\omega,t_2}(\mathbb{X}|\mathbb{BB}, I_{\omega,t_1}) - 9 = 1 \tag{5}$$

The mathematical incoherence of these terms is central to List and Pivato's defense of the emergent chance thesis. Thus, List and Pivato are committed to the view that, on the coarsening described in (4), the value of the information about which shaft the miners are in cannot be calculated, since the question of which shaft the miners are in is not answered by specifying any element of the coarse-grained algebra. This shows that List and Pivato's account of coarsening, which is central to their argument for the emergent chance thesis, is also liable to produce event algebras that are expressively impoverished in important ways.

By contrast, my account of coarsening is not susceptible to this worry. Recall that the coarsening function $\phi(\cdot)$ is a mapping from the algebra of events $\mathcal{A}_\Omega$ into itself, such that for any $E \in \mathcal{A}_\Omega$, $E \subseteq \phi(E)$. Let the set of worlds Ω be defined as the set of mappings of times into the seven-element set of fine-grained states specified above. A coarsening function playing the same representational role as the function defined in equation (4) can be defined as follows.

$$\phi(E) = \{\omega : \forall s,\, \omega(t_2) = s \text{ if and only if there exists some } \omega' \in E \text{ such that } \omega'(t_2) = s\} \tag{6}$$

In other words, $\phi(\cdot)$ maps any set of possible worlds E to the set of all and only those possible worlds $\phi(E)$ that agree with some world in E as to the state of the world at t_2. This ensures that if E is a set of worlds such that the miners are definitively in shaft A, or E is a set of worlds such that the miners are definitively in shaft B, but wherein it is not specified which shaft is blocked or whether the miners drown, then E is mapped to a set of worlds such that the miners are in either shaft A or shaft B. When coarsening is defined in this way, there is no conceptual problem in conditionalizing on the event that the miners are definitely in shaft A or definitely in shaft B, even though neither event is in the range of the coarsening function $\phi(\cdot)$. This is because these events are still in the algebra over which probability functions are defined. Thus, on my account of coarsening, we can represent the functional relationship between more fine and coarse-grained descriptions of the scenario, while retaining the ability to coherently calculate the maximum cost that rescuers should be willing to take on in order to determine which shaft contains the miners.

This issue with List and Pivato's defense of the emergent chance thesis generalizes beyond the Miner's Puzzle. In many cases, it may be that valuable information is elided when we coarsen the set of possible worlds over which events are defined. From the perspective of these more coarse-grained algebras, it is impossible to coherently say that this information is valuable or that it is missing from our picture of the world. By contrast, my account of coarsening will never run the risk of rendering unintelligible our judgments about the information that is missing from a more coarse-grained picture of the world.

4.4 Further Advantages: Simpson's Paradox

Some additional cases that demonstrate the advantages of my account of coarsening are instances of Simpson's Paradox.[9] In general, Simpson's Paradox occurs whenever correlations at one level of description are reversed at a coarser or finer level of description.[10] To illustrate, consider a

[9]Similarities between Simpson's Paradox and the Miners Puzzle have been noted by Kotzen (2013).

[10]Although Simpson's Paradox is so-named due its discussion in Simpson (1951), the earliest known discussion of the phenomenon is in Yule (1903). The beginning of philosophical attention to Simpson's Paradox is usually traced

Population	Phenotype	t_1	t_2	Average Offspring
1	Producer	150	630	4.2
	Non-Producer	50	270	5.2
2	Producer	100	270	2.7
	Non-Producer	100	330	3.3
3	Producer	50	60	1.2
	Non-Producer	150	240	1.6
Total	Producer	300	960	3.2
	Non-Producer	300	840	2.8

Table 2: Evolutionary Dynamics of *Escherichia coli* Populations

case in evolutionary biology, which is described in Chuang et al. (2009).[11] Within a population of *Escherichia coli* microbes, individual microbes can either produce or not produce Rhl autoinducer molecules; whether or not a given microbe is a producer of the autoinducer is genetically determined and passed down from a parent microbe to its offspring. These autoinducer molecules improve every microbe in a spatially isolated sub-population's ability to survive and reproduce; one need not be a producer to reap the advantages. However, production of the autoinducer is costly to any given microbe. As such, the best-case scenario for any given microbe is to be a non-producer in a population that is full of producers. The hypothetical evolution of three sub-populations of *Escherichia coli* is described in Table 2.[12] It is clear from the table that in each sub-population, non-producers of the autoinducer have an evolutionary advantage over producers. However, in the total population, it appears that producers have an evolutionary advantage over non-producers.

This example shows how in cases of Simpson's Paradox, we see a divergence in the truth values of what Fitelson (2017) calls the "suppositional" and "conjunctive" claims about the relationship between population, autoinducer phenotype, and fitness. To illustrate, consider the following two claims:

Suppositional: If a microbe is in a given sub-population j, then being a non-producer

to Cartwright (1979). See Malinas and Bigelow (2016) for a summary of Simpson's Paradox and its philosophical significance. It is worth noting that the term 'Simpson's Paradox', though common in the literature, is understood to be a misnomer; strictly speaking, there is no semantic paradox in cases where correlations are reversed at different levels of description.

[11] Similar cases of Simpson's Paradox in evolutionary biology are highlighted by Sober and Wilson (1998).

[12] The expected offspring calculations assume that all microbes die and are replaced by their offspring in the transition from t_1 to t_2.

is a more fitness-enhancing strategy than being a producer.

Conjunctive: Being a non-producer and being in sub-population j is a more fitness-enhancing strategy than being a producer in any sub-population.

It should be clear from the data that the suppositional claim is true, but the conjunctive claim is false. In each of the three sub-populations, non-producers of the autoinducer have a greater number of expected offspring than producers; thus, the suppositional claim is true. However, the conjunctive claim is false. For instance, a non-producer microbe in sub-population 3 has lower fitness than a producer microbe in sub-population 1.

If one cannot articulate the distinction between the suppositional and the conjunctive claim, and can instead only evaluate the conjunctive claim, then one loses the ability to make an important distinction about the strategic effectiveness of being a producer or a non-producer. When one sees that the suppositional claim is true while the conjunctive claim is false and assumes that an individual microbe's being a producer or non-producer does not cause other microbes to be a producer or non-producer, one concludes that a given microbe should adopt the strategy of being a non-producer; conditional on the microbe being in any sub-population, non-production is the best strategy. But if we cannot distinguish microbes by sub-population, then we may be misled into believing the claim that being a producer improves an organism's fitness more than being a non-producer. After all, the apparent truth of the conjunctive claim is supported by the data in the row of Table 2 labelled 'Total'. To believe such a claim would be to hold a fundamentally false belief about what would be evolutionarily advantageous to any given microbe in the population. In what follows, I will show that List and Pivato's coarsening method allows us to coarsen the set of possible worlds in a way that would lead us to form just this false belief.

To begin, consider a set of worlds Ω such that each world is a mapping from times t_1 and t_2 to a set of states S such that each state is represented by an ordered triple of ordered pairs $s_i = \langle\langle p_1, np_1\rangle, \langle p_2, np_2\rangle, \langle p_3, np_3\rangle\rangle$ where each p_j and np_j is an integer denoting the number of producers and non-producers in sub-population j. Thus, at time t_1 the population described in Table 2 is in state $s_i = \langle\langle 150, 50\rangle, \langle 100, 100\rangle, \langle 50, 150\rangle\rangle$, and at time t_2 the population is in the state $s_i = \langle\langle 630, 270\rangle, \langle 270, 330\rangle, \langle 60, 240\rangle\rangle$. Note that Table 2 represents the evolution of one possible world in Ω from t_1 to t_2. Let $F_{np,j>p,k}$ be the set of worlds such that the growth rate for non-producers from t_1 to t_2 in a given sub-population j is greater than the growth rate for producers in a given sub-population k over the same time period (allowing for the possibility that $j = k$). Let $F_{p,j>np,k}$ be the set of worlds such that the growth rate for producers from t_1 to t_2 in a given sub-population j is greater than the growth rate for non-producers in a given sub-population k over the same time period. This allows us to state the following, probabilistic versions of the suppositional and conjunctive claims:

> **Probabilistic Suppositional**: For all worlds ω and sub-populations j, it is the case that $p_{\omega,t_1}(F_{np,j>p,j}) > p_{\omega,t_1}(F_{p,j>np,j})$.
>
> **Probabilistic Conjunctive**: For all worlds ω and all sub-populations j and k, it is the case that $p_{\omega,t_1}(F_{np,j>p,k}) > p_{\omega,t_1}(F_{p,j>np,k})$.

In other words, the probabilistic suppositional claim states that at t_1, non-producers are more likely than producers to have a higher growth rate from t_1 to t_2 in any sub-population j. If the evolutionary dynamics instantiated in Table 2 hold in general for Ω, then this claim is true. By contrast, the probabilistic conjunctive claim states that at t_1, it is more likely than not that the growth rate from t_1 to t_2 for non-producers in a sub-population j is greater than the growth rate for producers from t_1 to t_2 in some other sub-population k. If we take the evolutionary dynamics in the table above to generalize across Ω, then this claim is false.

On List and Pivato's account of coarsening, some coarsenings of this set of possible worlds produce an algebra such that we cannot distinguish between the suppositional and conjunctive claims regarding the evolutionary advantage of being a non-producer. In particular, suppose that we adopt the following coarsening function:

$$\sigma(s_i = \langle\langle p_1, np_1\rangle, \langle p_2, np_2\rangle, \langle p_3, np_3\rangle\rangle) = \mathbb{s}_i = \langle p_1 + p_2 + p_3, np_1 + np_2 + np_3\rangle \tag{7}$$

In other words, $\sigma(\cdot)$ takes as its input a fine-grained state specifying the number of producers and non-producers in each population, and returns a coarse-grained state specifying the total population of producers and non-producers. As such, this coarsening yields a coarse-grained algebra that does not allow us to distinguish between the growth rates of microbes in any given sub-population. As the comparisons in the growth rates of various sub-populations of microbes are crucial to distinguishing between the suppositional and conjunctive claims, the coarsening function stated in equation (7) limits our ability to express crucial aspects of the system under study. Instead, we can only express the claim that in a given microbe population, the growth rate in producers from t_1 to t_2 is likely to be greater than the growth rate in non-producers, a claim that the data in the row of Table 2 labelled 'Total' would support. As argued above, this claim is deeply misleading.

As implied by Proposition 1, my account of coarsening can generate a coarsening function with a range that is related by a bijection to the set of possible events generated by the coarsening function $\sigma(\cdot)$, as described in equation (7). This function can be described as follows:

$$\phi(E) = \{\omega : \forall t, \omega(t) = \langle\langle p_1, np_1\rangle, \langle p_2, np_2\rangle, \langle p_3, np_3\rangle\rangle \text{ if and only if there exists an } \omega' \in E \text{ such that } \omega'(t) = \langle\langle p_4, np_4\rangle, \langle p_5, np_5\rangle, \langle p_6, np_6\rangle\rangle \text{ where } p_1 + p_2 + p_3 = p_4 + p_5 + p_6 \text{ and } np_1 + np_2 + np_3 = np_4 + np_5 + np_6\} \quad (8)$$

That is, $\phi(\cdot)$ maps each event to the union of the set of events that agree with it with respect to the total number of producers and non-producers in all worlds and times. Thus, the coarsening function $\phi(\cdot)$ represents the same supervenience relations between the evolution of the microbe sub-populations and the total microbe population as the coarsening function $\sigma(\cdot)$. However, because $\phi(\cdot)$ is a mapping of the fine-grained algebra of events into itself, we are still able to assign probabilities to events that are distinguished by the composition of particular sub-populations of microbes. As demonstrated above, this is not possible when the coarse-grained algebra of events is produced using the coarsening function $\sigma(\cdot)$. Thus, my account of coarsening does not threaten our ability to represent important distinctions in cases of Simpson's paradox, whereas List and Pivato's does.

5 Response to Counterarguments

An initial line of response to my arguments above could run as follows. In considering both the Miners Puzzle and Simpson's Paradox, I have defined List and Pivato-style coarsening functions that undercut our ability to articulate important aspects of either case, and taken this as evidence for the deficiency of their approach. However, rather than taking these examples to be bad results for List and Pivato's account of coarsening, why not simply take them to be bad results for the particular choice of coarsening function? The success of the special sciences, this argument continues, is due at least in part to the fact that those sciences coarsen the physical description of the world in the right way. Thus, in contrast with the examples that I have described above, in well-formulated special sciences we are able to express all important nuances of all relevant cases. According to this argument, the coarsening functions used in these special sciences will be just those that are able to express the important details of all domain-specific cases, while still using a coarse-grained event algebra.

In response, I argue that List and Pivato's coarsening method lacks the resources to articulate whether or not a given coarsening is deficient in the ways described above. Consider the case of the Miner's Puzzle. If we coarsen the set of possible worlds so as to elide information about which shaft the miners are in, then we lose the ability to say that this is information that we would like to have, if we could get it. One can tell a similar story with respect to the example of Simpson's Paradox that I have given above; by coarsening the set of possible worlds so as to elide information about the sub-population that the organism is in, we end up thinking that it is better for a microbe to produce the autoinducer than to not produce it. Further, from within this coarsening, there is no way to say that the coarsening is in any way deficient, or that sub-population-level statistics on the fitness of microbes would be valuable to a microbe that could understand them. In summary, absent some meta-algebra representing different possible coarsenings, List and Pivato's formalism lacks any way of explicitly comparing coarsenings. Further, even if such a meta-algebra were to be supplied, it is unclear what would be gained from such an algebra that one does not already get from the more parsimonious option of representing coarsening as a mapping from the event algebra into itself.

A second line of objection to what I have presented here stresses the fundamental motivation behind the emergent chance thesis. This argument proceeds as follows. At the level of description used in any given special science, events may be assigned non-extreme probabilities, and those probabilities may not be explained by any agent's lack of information about the world *as described by that special science*. These probabilities, the objection goes, deserve the name "objective chances" within the context of a given special science. List and Pivato's formal framework allows this to be articulated, whereas my account seems to imply that the only non-extreme probabilities that qualify as objective chances are those events that have non-extreme probability, conditional on a maximally fine-grained description of the world up to some relevant time.

In response, I would argue that my approach does leave open the possibility of a kind of special science objective chance, albeit one that is conceived of in a different way than List and Pivato have in mind. To illustrate, consider the example of a wheel of fortune that can stop on white or red. Suppose that the color that the wheel stops on is completely determined by the amount of force with which the wheel is spun. For instance, if the wheel is spun with anywhere from 14.0 to 14.1 Newtons of force, then it will land red, if it is spun with anywhere from 14.1 to 14.2 Newtons of force, then it will land white, and so on. If the wheel is spun at t_1, and I_{ω,t_1} specifies the force with which the wheel is spun, then $p_{\omega,t_1}(R|I_{\omega,t_1}) = 1$ or 0 for all ω. As such, if the probability assigned to the event that the wheel stops on red with any given spin is non-extreme, then it is purely epistemic. However, suppose that, as a matter of fact, actual spins of the wheel vary in force such that the wheel stops on red about half the time. Let $\phi(I_{\omega,t_1})$ be the event that the wheel is spun, with any force, at time t_1. Under these conditions, $p_{\omega,t_1}(R|\phi(I_{\omega,t_1})) = .5$. In the sense articulated by List and Pivato, this probability is purely epistemic, since it reflects our ignorance about the amount of force applied in any given spin. However, one could argue that the probability is still objective in the sense that it reflects a mind-independent fact about the distribution of possible amounts of force exerted in any given spin of the wheel. In the same way, if events in the special sciences are realized by microphysical conditions in a way that gives rise to non-extreme probabilities at a coarse-grained level of description, then perhaps they can also be thought of as objective in some sense, even if they are purely epistemic by List and Pivato's lights. A fully-fledged defense of this proposal would require substantially more argumentation, but I have outlined the proposal here so as to highlight the fact that rejecting List and Pivato's approach does not necessarily undermine our ability to treat special-science probabilities as objective in some sense.

For examples of proposals for understanding special-science probabilities that point in this direction, see Shalizi and Moore (2003), Lyon (2011), and Strevens (2011).[13] Alternatively, one could argue, along the lines of Ismael (2009, 2011) and Glynn (2010), that the very concept of what counts as an objective chance is relativized to to the conditional context in which the probability is assigned, so that an extreme objective chance for an event E conditional upon the world's microstate is perfectly compatible with a non-extreme chance for E conditional upon its macrostate.

[13]Though note the objections to Strevens (2011) in Canson (forthcoming).

In a similar vein, Hemmo and Shenker (2012, 2019) and Shenker (2017) have argued that it is an objective fact about certain measuring devices in statistical mechanics that these devices are sensitive to specific macrostates of a given system (see specifically Hemmo and Shenker 2019, p. 466). Thus, they argue, statistical mechanical systems are, in virtue of the measuring devices used to study them, objectively partitioned according to a particular coarse-graining, such that the non-extreme probability distributions over these coarse-grained states deserve the title of objective chances. What all of these proposals have in common is that, in contrast with List and Pivato's approach, they do not argue for the emergence of chances by appealing to the putative mathematical inconsistency of conditionalizing on the micro-state of the world when assigning conditional probabilities to macro-level events.

An additional potential counterargument to my position is inspired by an argument in List (2019, pp. 873-874). I reconstruct it here as follows. Suppose that Ω has countably many elements, and $\mathcal{A}_\Omega$ has uncountably many elements (for example, Ω might have the cardinality of the integers and $\mathcal{A}_\Omega$ might have the cardinality of the power set of the integers). This means that any language $\mathbf{L}$ with a countable vocabulary will not have terms to denote most elements of $\mathcal{A}_\Omega$. Next, let $\sigma(\cdot) : S \to \mathbb{S}$ be a coarsening function on the state space that comprises the co-domain of each possible world in Ω. Suppose further that applying the coarsening $\sigma(\cdot)$ yields a finite set of possible worlds $\mathbb{\Omega}$, such that $\mathbb{A}_{\mathbb{\Omega}}$ is also finite. In this scenario, $\varphi \in \mathbb{A}_{\mathbb{\Omega}}$ can be such that the set of possible worlds in Ω that are consistent with φ is not denoted by any element of $\mathbf{L}$, because $\mathbf{L}$ has only countably many elements, whereas $\mathcal{A}_\Omega$ has uncountably many. While List and Pivato's approach can represent such a coarsening, one could argue that mine cannot. This is because the function $\phi(\cdot) : \mathcal{A}_\Omega \to \mathcal{A}_\Omega$ cannot be expressed, since the inverse image $\phi^{-1}(\varphi)$ may not be an element of the algebra on the subsets of $\mathcal{A}_\Omega$ that are expressible by elements of the vocabulary of $\mathbf{L}$.

In response, I need only note that it is possible for a function to be well-defined when it is not the case that all elements of its domain are expressible according to a language with a countably large vocabulary. Indeed, any and all functions defined on the real numbers have just this property. Thus, while List's argument above may serve to establish that in many cases, the vocabulary of terms used to denote elements of higher-level algebra $\mathbb{A}_{\mathbb{\Omega}}$ cannot be reduced to expressible elements of a lower-level language with countably many terms in its vocabulary, this does not establish that a coarsening function of the kind that I propose here cannot be well-defined in such cases.

6 Conclusion

My aim in this paper has been to show that one argument for the emergent chance thesis can be blocked. As for the more general question of whether the probabilities typically used in the special sciences are genuine emergent chances, I believe that this debate mostly amounts to a verbal dispute about what counts as an objective chance. If an objective chance is a non-extreme probability that does not reflect any ignorance about the state of the world up until some time, then I would claim, in contrast with List and Pivato, that there are only objective chances if the underlying microphysics is indeterministic. However, if objective chances are just those probabilities that play an important role in the special sciences, then clearly such objective chances exist, regardless of our preferred formalization of the relationship between coarse-grained and fine-grained events.

Importantly, even if it turns out that special science probabilities are purely epistemic, it does not follow from this that they are not interesting in their own right. In studying the inter-theoretic relations between various sciences, we discover which fine-grained facts about a given target system can be systematically ignored, while still retaining all domain-relevant information. For instance, in studying the relationship between thermodynamics and statistical mechanics, we discover that if we only care about the thermodynamic properties of that gas, then specific information about the velocities of each particle in a box of gas can be ignored. These facts about which pieces of information get included in more coarse-grained descriptions of the world are highly informative as to the nature of the subject matter studied by these more coarse-grained theories. This is true independently of whether the probabilities included in those special science theories are objective chances, and independently of any particular formal representation of coarse-graining.

7 Appendix

7.1 Proof of Proposition 1

Proof. Let $\omega \in \Omega$ be any possible world. Recall that ω is a mapping $T \to S$. Let a "conjoined pair" of states $(s_i, s_j) \in S$ be any pair of states such that, according to the coarsening function $\sigma(\cdot)$, $\sigma(s_i) = \sigma(s_j) = \mathbb{s}$. Let $\Omega'_k \subseteq \Omega$ be a set of possible worlds such that, for any pair of worlds $(\omega_l, \omega_m) \in \Omega'_k$ and all t, the states $\omega_l(t)$ and $\omega_m(t)$ are conjoined. The set of such sets of worlds

$\Omega^\dagger = \{\Omega'_1, \Omega'_2, ..., \Omega'_n\}$ fully partitions the set of all possible worlds Ω. If the coarsening function $\sigma(\cdot)$ is applied to the set of states S, then each set of fine-grained worlds Ω'_k is replaced by a single coarse-grained world ω. Thus, we can define a bijection from $\Omega^\dagger$ into the set of coarse-grained possible worlds $\mathbb{\Omega}$, and therefore a bijection from $\mathcal{A}_{\Omega^\dagger}$ into $\mathbb{A}_{\mathbb{\Omega}}$.

Finally, define a coarsening function $\phi(\cdot) : \mathcal{A}_\Omega \to \mathcal{A}_\Omega$ such that $\mathcal{A}_{\Omega^\dagger}$ is the range of $\phi(\cdot)$, i.e. $\Gamma_{\phi(\cdot)} = \mathcal{A}_{\Omega^\dagger}$. It follows that there is a bijection between $\Gamma_{\phi(\cdot)}$ and $\mathbb{A}_{\mathbb{\Omega}}$. For any $E \in \mathcal{A}_\Omega$ and any $\phi(E) \in \mathcal{A}_{\Omega^\dagger}$, if $\omega \in E$, then $\omega \in \phi(E)$, since any ω is conjoined with itself. Thus, in keeping with my proposed restriction on any coarsening function $\phi(\cdot)$, for any $E \in \mathcal{A}_\Omega$, $E \subseteq \phi(E)$. $\square$

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